

let's add fractions!

$$\frac{(x+2)}{(x+2)} \frac{1}{x} + \frac{1}{x+2} \frac{(x)}{(x)}$$

$$\frac{x+2+x}{x(x+2)} = \frac{2x+2}{x^2+2x}$$

SO ...

$$\left(\frac{1}{x} + \frac{1}{x+2} = \frac{2x+2}{x^2+2x} \right. \quad \begin{array}{l} du = 2x+2 \\ u = x^2+2x \end{array}$$

?

$$\ln x + \ln(x+2) =$$

← I want to break it down because it is easier.

6.5 Logistic Growth

In this section, we are learning how to break apart fractions, so that we could integrate them easier.

Ex. $\frac{3}{10} = \frac{1}{10} + \frac{1}{5}$

Handwritten notes:
An orange oval circles the numerators 1 and 1 in the partial fractions. An arrow points from the text "we will find this!" to the oval.
Below the denominator 10, the number "2 · 5" is written in red.

Partial Fraction Decomposition with Distinct Linear Denominators

If $f(x) = \frac{P(x)}{Q(x)}$, where P and Q are polynomials with the degree of P less than the degree of Q , and if $Q(x)$ can be written as a product of distinct linear factors, then $f(x)$ can be written as a sum of rational functions with distinct linear denominators.

EXAMPLE 1 Finding a Partial Fraction Decomposition

Write the function $f(x) = \frac{x - 13}{2x^2 - 7x + 3}$ as a sum of rational functions with linear denominators.

SOLUTION

Since $f(x) = \frac{x - 13}{(2x - 1)(x - 3)}$, we will find numbers A and B so that

$$f(x) = \frac{A}{2x - 1} + \frac{B}{x - 3} = \frac{x - 13}{(2x - 1)(x - 3)}$$

$$A(x - 3) + B(2x - 1) = x - 13. \quad (1)$$

Setting $x = 3$ in equation (1), we get

$$A(0) + B(5) = -10, \text{ so } B = -2.$$

Setting $x = \frac{1}{2}$ in equation (1), we get

$$A\left(-\frac{5}{2}\right) + B(0) = -\frac{25}{2}, \text{ so } A = 5.$$

$$\text{Therefore } f(x) = \frac{x - 13}{(2x - 1)(x - 3)} = \frac{5}{2x - 1} - \frac{2}{x - 3}.$$

Now try Exercise 3.

In Exercises 1–4, find the values of A and B that complete the partial fraction decomposition.

1. $\frac{x-12}{x^2-4x} = \frac{A}{x} + \frac{B}{x-4}$ $A=3, B=-2$

2. $\frac{2x+16}{x^2+x-6} = \frac{A}{x+3} + \frac{B}{x-2}$ $A=-2, B=4$

3. $\frac{16-x}{x^2+3x-10} = \frac{A}{x-2} + \frac{B}{x+5}$ $A=2, B=-3$

4. $\frac{3}{x^2-9} = \frac{A}{x-3} + \frac{B}{x+3}$ $A=1/2, B=-1/2$

LCM: $x(x-4)$

$$\frac{x-12}{x^2-4x} = \frac{A}{x} + \frac{B}{x-4}$$

$$x-12 = A(x-4) + Bx$$

$x=4$;

$$4-12 = A(4-4) + B(4)$$

$$-8 = 4B$$

$$B = -2$$

$x=0$;

$$0-12 = A(0-4) + B(0)$$

$$-12 = -4A$$

$$A = 3$$